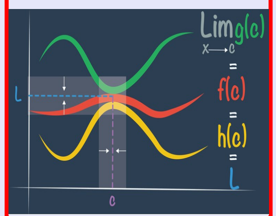
# Math 261

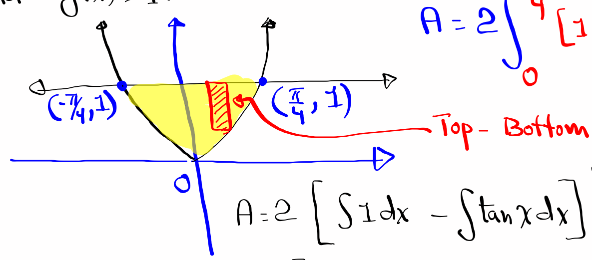
# Fall 2022

# Lecture 48




Feb 19-8:47 AM

Consider the enclosed region by $f(x) = |\tan x|$ and $g(x) = 1$.




$$A = 2\int_0^{\frac{\pi}{4}} [1 - \tan x]\,dx$$

$$A = 2\left[\int 1\,dx - \int \tan x\,dx\right]\Big|_0^{\pi/4}$$

$$A = 2\left[x - \int \tan x\,dx\right]\Big|_0^{\pi/4}$$

$$= 2\left[x - (-\ln|\cos x|)\right]\Big|_0^{\pi/4}$$

$$= 2\left[x + \ln(\cos x)\right]\Big|_0^{\pi/4}$$

$$= 2\left[\frac{\pi}{4} + \ln\cos\frac{\pi}{4} - 0 - \ln\cos 0\right]$$

$$= \boxed{2\left[\frac{\pi}{4} + \ln\frac{\sqrt{2}}{2}\right]}$$

$\int \tan x\,dx$ Calc 2

$= \int \frac{\sin x}{\cos x}\,dx$

Let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$= \int \frac{-1}{u}\,du$

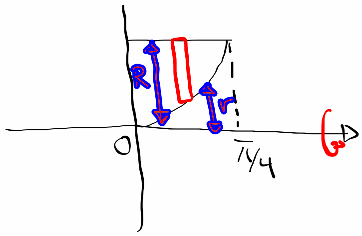
$\int \frac{1}{u}\,du = \ln|u| + C$

$= -\ln|u| + C$

$= -\ln|\cos x| + C$

Nov 23-8:57 AM

Rotate the region by $x$-axis, and find its volume.




$$V = 2\int_0^{\pi/4} \pi\left[1^2 - \tan^2 x\right]dx$$

$$= 2\pi\int_0^{\pi/4}\left[1 - \tan^2 x\right]dx$$

$$= 2\pi\int_0^{\pi/4}\left[1 - \sec^2 x + 1\right]dx$$

$$= 2\pi\int_0^{\pi/4}(2 - \sec^2 x)\,dx$$

$$= 2\pi\left[2x - \tan x\right]\Big|_0^{\pi/4}$$

$$= 2\pi\left[2\cdot\frac{\pi}{4} - \tan\frac{\pi}{4} - 0\right]$$

$$= 2\pi\left[\frac{\pi}{2} - 1\right] = \boxed{\pi^2 - 2\pi}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{\sin^2 x}{\cos^2 x} + 1 = \frac{1}{\cos^2 x}$$

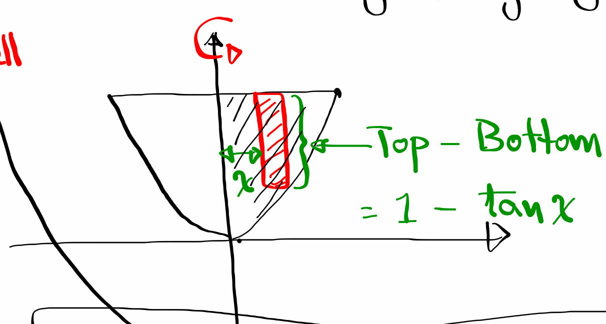
$$\tan^2 x + 1 = \sec^2 x$$

$$\tan^2 x = \sec^2 x - 1$$

Nov 23-9:08 AM

Now rotate the region by $y$-axis. **Set-up** the integral for the Volume.

Shell

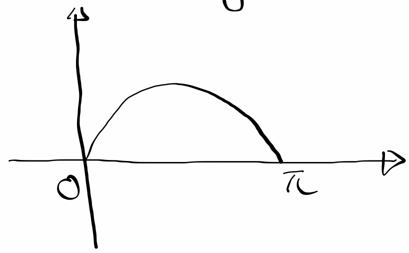



$$\boxed{V = \int_0^{\pi/4} 2\pi\cdot x\cdot(1 - \tan x)\,dx}$$

Nov 23-9:15 AM

Consider the region bounded by $y = \sin x$ in QI.

Set-up only.

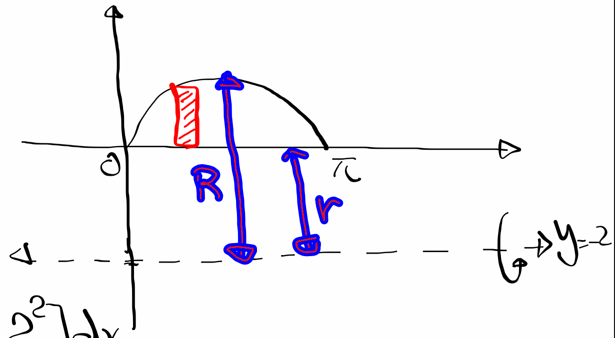



Volume if we rotate about $y = -2$.

washer method

$R = 2 + \sin x$

$r = 2$




$$V = \pi \int_0^{\pi} \left[(2+\sin x)^2 - 2^2\right] dx$$

Nov 23-9:20 AM

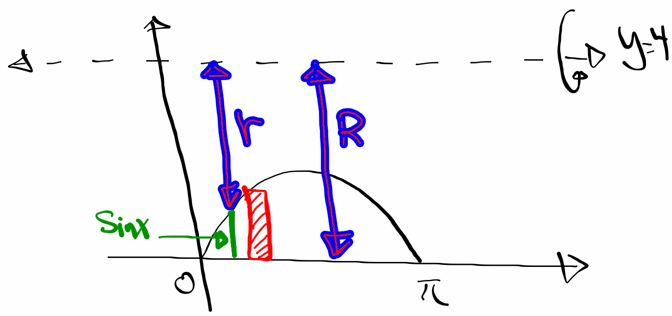
Let's rotate about $y = 4$. Set-up only

Find Volume

washer

$R = 4$

$r = 4 - \sin x$



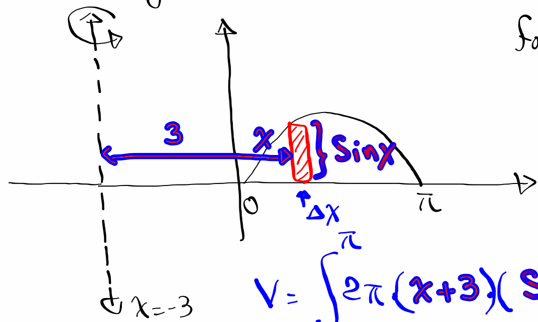

$\sin x + r = 4$

$r = 4 - \sin x$

$$V = \pi \int_0^{\pi} \left[4^2 - (4 - \sin x)^2\right] dx$$

Nov 23-9:25 AM

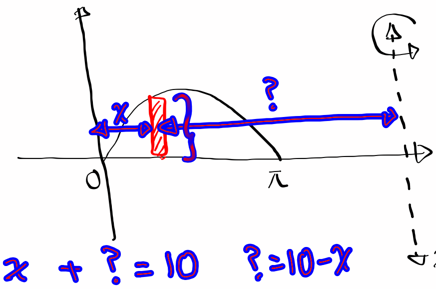
Rotate the region about $x=-3$. Set-up only for volume.




Shell Method

$$V=\int_0^{\pi} 2\pi(x+3)(\sin x)\,dx$$

Now rotate about $x=10$. Set-up only for Volume.




Shell Method

$$V=\int_0^{\pi} 2\pi(10-x)(\sin x)\,dx$$

$x + ? = 10 \quad ? = 10 - x$

Nov 23-9:29 AM

Find $f_{ave}$ for $f(x)=\sec^2\frac{x}{2}$ on $[0,\pi/2]$

$$f_{ave}=\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$=\frac{1}{\frac{\pi}{2}-0}\int_0^{\pi/2}\sec^2\frac{x}{2}\,dx=\frac{2}{\pi}\int_0^{\pi/2}\sec^2\frac{x}{2}\,dx$$

$u=\frac{x}{2}$ $\quad x=0 \quad u=0$

$du=\frac{1}{2}dx$ $\quad x=\frac{\pi}{2} \quad u=\frac{\pi}{4}$

$2du=dx$

$$=\frac{2}{\pi}\int_0^{\pi/4}\sec^2 u\cdot 2du$$

$$=\frac{4}{\pi}\left[\tan u\Big|_0^{\pi/4}\right]$$

$$=\frac{4}{\pi}\left[\cancel{\tan\frac{\pi}{4}}^{1}-\cancel{\tan 0}^{0}\right]$$

$$=\boxed{\frac{4}{\pi}}$$

Nov 23-9:36 AM

find $c$ in $(0,2)$ such that $f(c) = f_{ave}$ on $[0,2]$

for $f(x) = \dfrac{2x}{(1+x^2)^2}$.

.22
&
1.21

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2-0}\int_0^2 \frac{2x}{(1+x^2)^2}\,dx$$

$u = 1+x^2$

$du = 2x\,dx$

$$= \frac{1}{2}\int_1^5 \frac{1}{u^2}\,du$$

$$= \frac{1}{2}\int_1^5 u^{-2}\,du = \frac{1}{2}\cdot\frac{u^{-1}}{-1}\Big|_1^5 = \frac{-1}{2}\cdot\frac{1}{u}\Big|_1^5$$

$$= \frac{-1}{2}\left[\frac{1}{5} - 1\right] = \frac{-1}{2}\cdot\frac{-4}{5} = \boxed{\frac{2}{5}}\ f_{ave}$$

$f(c) = f_{ave}$

$$\frac{\cancel{2}c}{(1+c^2)^2} = \frac{\overset{1}{\cancel{2}}}{5}$$

$(1+c^2)^2 = 5c$

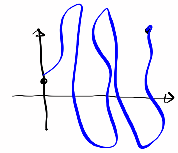
$1 + 2c^2 + c^4 = 5c$

$c^4 + 2c^2 - 5c + 1 = 0$

$g(c) = c^4 + 2c^2 - 5c + 1$

$g(0) = 1$

$g(2) = 15$



$c \approx .22$

$c \approx 1.21$

Nov 23-9:42 AM